A prediction of neutrino mixing matrix with CP violating phase

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Abstract

The latest experimental progress have established three kinds of neutrino oscillations with three mixing angles measured to rather high precision. There is still one parameter, i.e., the CP violating phase, missing in the neutrino mixing matrix. It is shown that a replay between different parametrizations of the mixing matrix can determine the full neutrino mixing matrix together with the CP violating phase. From the maximal CP violation observed in the original Kobayashi-Maskawa (KM) scheme of quark mixing matrix, we make an Ansatz of maximal CP violation in the neutrino mixing matrix. This leads to the prediction of all nine elements of the neutrino mixing matrix and also a remarkable prediction of the CP violating phase $\delta_{\rm CK} = (85.48^{+4.67(+12.87)}_{-1.80(-4.90)})^\circ$ within 1σ (3σ) range from available experimental information. We also predict the three angles of the unitarity triangle corresponding to the quark sector for confronting with the CP-violation related measurements.

Key words: CP violating phase, neutrino, mixing matrix

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The recent measurements of the neutrino mixing angle θ_{13} by the T2K, MINOS and Double Chooz collaborations [1], especially the latest ones by the Daya Bay Collaboration [2] and the RENO Collaboration [3], have led to the establishment of three kinds of neutrino oscillations. The three mixing angles, i.e., θ_{12}, θ_{23}, and θ_{13}, have been measured to rather high precision, and there have been some perspectives [4,5,6,7,8,9] by these novel experimental progress. As the three mixing angles are sizable, the neutrino physics has entered an era of precise measurement. A promising chance is viable for the measurement of the CP violating phase δ in future experiments. It is thus timely to look at the CP violating phase from theoretical aspects [7,8,9].

The mixing of fermions is a significant feature of fundamental particles, i.e., of quarks and leptons, and the mixing is well described by fermion mixing matrices [10,11,12]. The mixing of quarks is described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [11,12], with all parameters, i.e., three mixing angles and one CP violating phase, determined to rather high precision experimentally. The misalignment of the flavor eigenstates with the mass eigenstates in the lepton sector is also described by a mixing matrix, namely the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [10]. The PMNS matrix is defined as $U_{\mathrm{PMNS}} = U_L^{l\dagger} U_L^{\nu}$ and can be expressed generally as

$$U_{\mathrm{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}. \tag{1}$$

In the representation that the mass matrix of charged leptons is diagonal, the PMNS matrix represents the neutrino mixing, therefore we can also call it the neutrino mixing matrix. In case the neutrinos are of Dirac type, the neutrino mixing matrix can be parameterized by three rotation angles and a CP violating phase. Two additional phase angles are needed for the PMNS matrix if the neutrinos are of Majorana type. For the neutrino mixing, the Majorana phase angles do not affect the absolute values of the elements of mixing matrix and are omitted in the following discussion.

There are many possible ways to parameterize the mixing matrix in terms of four independent parameters. One of such parametrizations is the Chau-Keung (CK) scheme [13] adopted by Particle Data Group [14,15,16] as the standard one, which is

$$U_{\rm CK} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CK}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CK}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CK}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CK}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CK}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CK}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CK}} & c_{23}c_{13} \end{pmatrix}. \quad (2)$$

Another well-discussed parametrization is the original Kobayashi-Maskawa (KM) scheme [12], which is,

$$U_{\rm KM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & e^{i\delta_{\rm KM}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & s_3 & -c_3 \end{pmatrix}$$
$$= \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta_{\rm KM}} & c_1c_2s_3 + s_2c_3e^{i\delta_{\rm KM}} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta_{\rm KM}} & c_1s_2s_3 - c_2c_3e^{i\delta_{\rm KM}} \end{pmatrix}. \quad (3)$$

It is rather interesting that this scheme allows for almost perfect maximal CP violation, i.e., the CP violating phase $\delta_{\rm KM}^{\rm quark} = 90^\circ$, for quarks [17,18,19,20,21], whereas in the standard parametrziation $\delta_{\rm CK}^{\rm quark} = 68.9^\circ$[16], which deviates from the maximal CP violation. This inspires us to make a prediction of the neutrino mixing matrix with all nine elements determined based on experimental information of three mixing angles together with an Ansatz of maximal CP violation for the KM-scheme of mixing matrix.

We should notice that the absolute values of the corresponding elements of the mixing matrix should be the same for different parametrizations, but the phase of each element may differ significantly. Also the degree of CP violation, such as whether it is maximal or minimal, is parametrization dependent. Most previous information of neutrino mixing matrix are expressed by parameters in the standard parametrization, and we still cannot combine the three measured mixing angles with the maximal CP phase in the KM-scheme in a direct way to predict the nine elements of the mixing matrix. It is thus necessary to make a replay between different schemes for a full prediction of the nine elements of the neutrino mixing matrix together with the CP violating phase $\delta_{\rm CK}$ in the standard parametrization.

The observables of the neutrino oscillation experiments are related to the

mixing angles of the standard parametrization. A global fitting of neutrino mixing angles based on previous experimental data and T2K and MINOS experiments (1σ (3σ)) [22] gives,

$$\sin^2\theta_{12} = 0.312, \quad 0.296 - 0.329(1\sigma), \quad 0.265 - 0.364(3\sigma); \tag{4}$$
$$\sin^2\theta_{23} = 0.42, \quad 0.39 - 0.50(1\sigma), \quad 0.34 - 0.64(3\sigma). \tag{5}$$

Combined with the latest result

$$\sin^2\theta_{13} = 0.024, \quad 0.020 - 0.028(1\sigma), \quad 0.010 - 0.038(3\sigma) \tag{6}$$

from the Daya Bay Collaboration [2], we can get five moduli of the PMNS matrix elements from the standard parametrization without knowledge of the CP violating phase. Notice that the error range for θ_{13} of the Daya Bay result is calculated by an assumption of Gaussian distribution, and the 1σ deviation is estimated by $\sigma^2 = \sigma^2_{\rm stat} + \sigma^2_{\rm syst}$. The five matrix elements are,

$$|U_{e1}| = c_{12}c_{13} = \sqrt{(1-s_{12}^2)(1-s_{13}^2)} = 0.8195^{+0.010(+0.032)}_{-0.010(-0.029)}; \tag{7}$$
$$|U_{e2}| = s_{12}c_{13} = \sqrt{s_{12}^2(1-s_{13}^2)} = 0.5518^{+0.015(+0.046)}_{-0.014(-0.042)}; \tag{8}$$
$$|U_{e3}| = |s_{13}| = \sqrt{s_{13}^2} = 0.1549 \pm 0.013(\pm 0.045); \tag{9}$$
$$|U_{\mu 3}| = s_{23}c_{13} = \sqrt{s_{23}^2(1-s_{13}^2)} = 0.6403^{+0.061(+0.168)}_{-0.023(-0.061)}; \tag{10}$$
$$|U_{\tau 3}| = c_{23}c_{13} = \sqrt{(1-s_{23}^2)(1-s_{13}^2)} = 0.7524^{+0.052(+0.143)}_{-0.020(-0.052)}. \tag{11}$$

With these five moduli, together with an Ansatz of maximal CP violation $\delta_{\rm KM} = 90^\circ$, we can get the mixing angles in the KM parametrization, which are,

$$\theta_1 = (34.97^{+1.00(+3.20)}_{-1.00(-2.90)})^\circ, \quad \theta_2 = (39.87^{+5.18(+14.21)}_{-1.97(-5.18)})^\circ, \quad \theta_3 = (15.68^{+1.34(+4.63)}_{-1.33(-4.60)})^\circ. \tag{12}$$

The corresponding trigonometric functions are,

$$\sin\theta_1 = 0.5732^{+0.014(+0.046)}_{-0.014(-0.042)}, \quad \cos\theta_1 = 0.8194^{+0.010(+0.032)}_{-0.010(-0.029)}; \tag{13}$$
$$\sin\theta_2 = 0.6411^{+0.069(+0.190)}_{-0.026(-0.069)}, \quad \cos\theta_2 = 0.7674^{+0.058(+0.159)}_{-0.022(-0.058)}; \tag{14}$$
$$\sin\theta_3 = 0.2703 \pm 0.021(\pm 0.078), \quad \cos\theta_3 = 0.9628 \pm 0.006(\pm 0.022). \tag{15}$$

Combined with the maximal CP violating phase $\delta_{\rm KM} = 90^\circ$ and the trigonometric functions in Eq.(3), we can get all the moduli of the PMNS matrix, which is,

$$|U_{\rm PMNS}| = \begin{pmatrix} 0.8195^{+0.010(+0.032)}_{-0.010(-0.029)} & 0.5518^{+0.015(+0.046)}_{-0.014(-0.042)} & 0.1549 \pm 0.013(\pm 0.045) \\ 0.4399^{+0.045(+0.129)}_{-0.024(-0.068)} & 0.6297^{+0.045(+0.123)}_{-0.018(-0.048)} & 0.6403^{+0.061(+0.168)}_{-0.023(-0.061)} \\ 0.3675^{+0.049(+0.140)}_{-0.024(-0.068)} & 0.5467^{+0.051(+0.143)}_{-0.021(-0.059)} & 0.7524^{+0.052(+0.143)}_{-0.020(-0.052)} \end{pmatrix}. \quad (16)$$

Then we can work out the CP violating phase in the standard parametrization, using the following expressions,

$$|U_{\mu 1}| = |-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CK}}|; \quad (17)$$
$$|U_{\mu 2}| = |c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CK}}|; \quad (18)$$
$$|U_{\tau 1}| = |s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CK}}|; \quad (19)$$
$$|U_{\tau 2}| = |-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CK}}|. \quad (20)$$

We can calculate $\delta_{\rm CK}$ from one of the above four equations. Using the original input from Eq. (4) to Eq. (6) in the process of calculating, we get the same central values of $\delta_{\rm CK}$ and the same error bars from the four equations given the effective digits we keep in the result. The resulting $\delta_{\rm CK}$ from the above expressions is

$$\delta_{\rm CK} = (85.48^{+4.67(+12.87)}_{-1.80(-4.90)})^\circ \quad (21)$$

within 1σ (3σ) range. The four elements of $|U_{\mu 1}|$, $|U_{\mu 2}|$, $|U_{\tau 1}|$ and $|U_{\tau 2}|$ at the left-lower corner of Eq. (16) are predictions of our analysis. The full neutrino mixing matrix predicted in Eq. (16) can be used to construct also the phase factors for all nine elements once a specific scheme of parametrization is chosen [4,6]. Our predictions of the full neutrino mixing matrix Eq. (16) together with the CP violating phase Eq. (21) in the standard parametrization can be conveniently applied for phenomenological analysis.

By the way, it is helpful to work out the Jarlskog invariant [23] in the two schemes of parametrizations above,

$$\mathcal{J}_{\rm CK} = \frac{1}{8}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\sin\delta_{\rm CK} = 0.0345^{+0.0029(+0.0100)}_{-0.0028(-0.0097)}; \quad (22)$$
$$\mathcal{J}_{\rm KM} = \frac{1}{8}\sin\theta_1\sin 2\theta_1\sin 2\theta_2\sin 2\theta_3\sin\delta_{\rm KM} = 0.0345^{+0.0030(+0.0101)}_{-0.0028(-0.0097)}, \quad (23)$$

which are consistent with each other. The value of this parameter is sizable than previous expectation and it is thus meaningful to design experiments for the measurement of the CP violating phase through neutrino oscillation processes. We notice that $\mathcal{J}_{\rm KM}$ possesses a maximal CP violation as implied from our Ansatz $\delta_{\rm KM} = 90^\circ$, whereas $\mathcal{J}_{\rm CK}$ is close to a maximal CP violation as a phenomenological consequence from our analysis.

The unitarity triangles constructed from the unitarity conditions $\Sigma_i U_{ij} U^*_{ik} = \delta_{jk}(j \neq k)$ and $\Sigma_j U_{ij} U^*_{kj} = \delta_{ik}(i \neq k)$ carry information on the CP violation [24,25]. Actually, in the quark sector, the CP violating information can be obtained from the observables α, β, γ, which are the inner angles of the db unitarity triangle. As is pointed out in Ref. [26], the possibility of reconstructing the unitarity triangle can be viewed as an alternative way in search for the CP violation in both the oscillation and nonoscillation experiments. It is worth mention that the unitarity triangles carry information of CP violation in a convention-independent way, and this makes them a better candidate in comparison with the CP violating phase δ as in any angle-phase parametrizations. As a result, it is worthwhile to calculate the inner angles of the $\nu_2\nu_3$ unitarity triangle,

$$U_{e2}U^*_{e3} + U_{\mu 2}U^*_{\mu 3} + U_{\tau 2}U^*_{\tau 3} = 0, \tag{24}$$

which is the correspondent of the db unitarity triangle in the lepton sector [27]. The result is,

$$\alpha = \varphi_2 = \arg(-\frac{U_{\tau 2}U^*_{\tau 3}}{U_{e2}U^*_{e3}}) = (78.58^{+4.15(+11.63)}_{-1.79(-5.28)})^\circ; \tag{25}$$

$$\beta = \varphi_1 = \arg(-\frac{U_{\mu 2}U^*_{\mu 3}}{U_{\tau 2}U^*_{\tau 3}}) = (12.00^{+1.84(+5.65)}_{-1.22(-4.10)})^\circ; \tag{26}$$

$$\gamma = \varphi_3 = \arg(-\frac{U_{e2}U^*_{e3}}{U_{\mu 2}U^*_{\mu 3}}) = (89.42^{+3.94(+10.85)}_{-1.53(-4.06)})^\circ. \tag{27}$$

As the unitarity triangle is convention-independent, we adopt the KM scheme parameters in Eq. [15] together with our Ansatz $\delta_{\rm KM} = 90^\circ$ as input for the complex PMNS matrix. The above result can be tested when the unitarity triangles can be reconstructed from future experiments related to the CP-violation of neutrinos.

For the ν_e appearance channel, i.e., one of the golden channels for leptonic CP violation, the oscillation probability is [28],

$$\begin{aligned} P(\nu_\mu \to \nu_e) &\approx \sin^2\theta_{23}\frac{\sin^2 2\theta_{13}}{(\hat{A}-1)^2}\sin^2((\hat{A}-1)\Delta) \\ &+\alpha\frac{\sin\delta_{CP}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}}{\hat{A}(1-\hat{A})}\sin(\Delta)\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta) \\ &+\alpha\frac{\cos\delta_{CP}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}}{\hat{A}(1-\hat{A})}\cos(\Delta)\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta) \\ &+\alpha^2\frac{\cos^2\theta_{23}\sin^2 2\theta_{12}}{\hat{A}^2}\sin^2(\hat{A}\Delta), \end{aligned} \tag{28}$$

where $\alpha = \Delta m_{21}^2/\Delta m_{31}^2$, $\Delta = \Delta m_{31}^2 L/4E$, $\hat{A} = 2VE/\Delta m_{31}^2$, and $V = \sqrt{2}G_F n_e$. n_e is the density of electrons in the Earth and $\hat{A}$ describes the strength of the matter effects. Measurements of this probability over different beam energies can impose constraints in the (θ_{13}, $\delta_{\rm CP}$) parameter space. It is pointed in Ref. [29] that combining 8 GeV and 60 GeV data makes it possible for a measurement of $\delta_{\rm CP}$ with an error of $\pm 10°$ at $\delta_{\rm CP} = 90°$.

Actually, it is pointed out that all three types of neutrino beams have the discovery potential for the CP violation given the relatively large θ_{13} [30]. Though the 3σ discovery region are limited to 25% of all possible values for $\delta_{\rm CP}$ in the upgraded T2K and NOvA experiments, the overall 3σ discovery reach of the Long-Baseline Neutrino Experiment (LBNE) can be around 70% of all possible values for $\delta_{\rm CP}$ [31]. Simulations [29] have shown that our prediction of $\delta_{\rm CP}$ lies in the range that can be directly examined in the Project-X of the LBNE.

It is interesting to notice that our procedure leads to a quasi-maximal CP violation in the standard parametrization, and such prediction differs from some theoretical expectations [7,8,9]. However, there have been some theoretical investigations indicating that a large CP violating phase δ_{CK} can be understood from some basic asymmetries. The near maximal CP violation with a large θ_{13} from our analysis is in accordance with a general approach based on residual Z_2 symmetries [32]. A maximal CP violation is also predicted from the octahedral symmetry for the family symmetry of the neutrino-lepton sector [33]. In Ref. [34], a prediction of $\delta_{\rm CK} = (60 - 90)°$ is made within the framework of discrete groups, i.e. A_4, S_4 and A_5. Actually, the range for $\delta_{\rm CK} = (60 - 90)°$ can be translated into $\delta_{\rm KM} = (60.31 - 90)°$ by noticing

$$\sin\delta_{\rm KM} = \frac{\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}}{\sin\theta_1\sin 2\theta_1\sin 2\theta_2\sin 2\theta_3}\sin\delta_{\rm CK} = 1.00311\sin\delta_{\rm CK}. \quad (29)$$

Thus our prediction of a quasi-maximal $\delta_{\rm CK}$ or a maximal $\delta_{\rm KM}$ can acquire the theoretical support from basic considerations.

In summary, we can predict the neutrino mixing matrix with all elements determined together with a prediction of the CP violating phase. We also predict the three angles of the unitarity triangle corresponding to the quark sector for confronting with the CP-violation related measurements. A similar exercise can be performed to the quark case, and we can get proved that the same procedure can also lead to a successful reproduction of the CP violating phase $\delta_{\rm CK}^{\rm quark}$ in the standard parametrization of the CKM mixing matrix. Our prediction is model independent without any ambiguity, except that the parameters can be also gotten from global fitting procedure instead of the analytic expressions adopted in this paper. We expect a test of our prediction of the full neutrino mixing matrix and the corresponding CP violating phase, or

the three angles of the unitarity triangle in a convention independent manner as in the quark case, through future experiments.

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